

DEVELOPMENT AND ANALYSIS OF A KINEMATIC-DYNAMIC MODEL OF A ROTATING-DISC MECHATRONIC SYSTEM

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Abstract. This study develops a kinematic and dynamic model of a rotating-disc mechatronic system designed to promote a uniform layer-by-layer distribution of microclimatic parameters during incubation. The computational configuration comprises a central shaft, horizontal discs, vertical supports, payload, and a DC motor. The motion cycle is divided into acceleration, constant-speed, and deceleration phases; the trapezoidal angular-velocity profile and the time variation of angular position are analyzed. The total moment of inertia of the rotating components is determined, and the Newton–Euler equation is formulated using the equivalent inertia reflected to the motor shaft through the gear transmission. The mechanical model is coupled with the electrical equation of the DC motor to derive transfer functions from the input voltage to angular velocity and angular position. The model is evaluated in terms of drive transient response, pole locations, stability, torque, and power requirements. The results provide a computational basis for implementing periodic 180° disc rotation in an energy-efficient cyclic operating mode and for selecting the drive parameters on a rational engineering basis.

Keywords: mechatronic system, rotating disc, kinematic model, dynamic model, moment of inertia, DC motor, gear transmission, transfer function, angular velocity, torque, stability.

Аннотация. В данном исследовании разработана кинематическая и динамическая модель мехатронной системы с вращающимися дисками, предназначенной для обеспечения равномерного послойного распределения параметров микроклимата в процессе инкубации. Расчётная схема включает центральный вал, горизонтальные диски, вертикальные опоры, полезную нагрузку и двигатель постоянного тока. Цикл движения разделён на фазы

разгона, движения с постоянной скоростью и торможения; проанализированы трапецеидальный профиль угловой скорости и изменение углового положения во времени. Определён суммарный момент инерции вращающихся компонентов, а также сформулировано уравнение Ньютона–Эйлера с использованием эквивалентного момента инерции, приведённого к валу двигателя через зубчатую передачу. Механическая модель связана с электрическим уравнением двигателя постоянного тока, на основе чего получены передаточные функции от входного напряжения к угловой скорости и угловому положению. Модель оценена с точки зрения переходной характеристики привода, расположения полюсов, устойчивости, требуемого крутящего момента и мощности. Полученные результаты создают расчётную основу для реализации периодического поворота дисков на 180° в энергоэффективном циклическом режиме работы, а также для рационального инженерного выбора параметров привода.

Ключевые слова: мехатронная система, вращающийся диск, кинематическая модель, динамическая модель, момент инерции, двигатель постоянного тока, зубчатая передача, передаточная функция, угловая скорость, крутящий момент, устойчивость.

Annotatsiya. Mazkur tadqiqotda inkubatsiya jarayonida mikroiklim parametrlarining qatlamlar bo'yicha bir tekis taqsimlanishini ta'minlashga mo'ljallangan aylanuvchi diskli mexatronik tizimning kinematik va dinamik modeli ishlab chiqilgan. Hisoblash sxemasi markaziy val, gorizontall disklar, vertikal tayanchlar, foydali yuk va o'zgarmas tok elektr dvigatelidan tashkil topgan. Harakat sikli tezlanish, o'zgarmas tezlikda harakatlanish va sekinlanish bosqichlariga ajratilgan; burchak tezligining trapetsiyasimon profili hamda burchak holatining vaqt bo'yicha o'zgarishi tahlil qilingan. Aylanuvchi qismlarning umumiy inersiya momenti aniqlanib, tishli uzatma orqali dvigatel valiga keltirilgan ekvivalent inersiya momentidan foydalangan holda Nyuton–Eyler tenglamasi shakllantirilgan. Mexanik model o'zgarmas tok dvigatelining elektr tenglamasi bilan bog'lanib, kirish

kuchlanishidan burchak tezligi va burchak holatiga o'tish funksiyalari hosil qilingan. Model yuritmaning o'tkinchi jarayoni, qutblarning joylashuvi, barqarorlik, aylantiruvchi moment va quvvat talablari nuqtayi nazaridan baholangan. Olingan natijalar disklarning davriy ravishda 180° ga aylanishini energiya tejamkor siklik ish rejimida amalga oshirish hamda yuritma parametrlarini muhandislik nuqtayi nazaridan oqilona tanlash uchun hisoblash asosini yaratadi.

Kalit so'zlar: mexatronik tizim, aylanuvchi disk, kinematik model, dinamik model, inersiya momenti, o'zgarmas tok dvigateli, tishli uzatma, uzatish funksiyasi, burchak tezligi, aylantiruvchi moment, barqarorlik.

INTRODUCTION. In incubation equipment, no uniform temperature and airflow distributions throughout the chamber volume make it difficult to provide identical environmental conditions for biological objects located at different levels. Therefore, a rotating-disc mechanism that periodically changes the spatial position of the trays can be regarded as an actuator that enhances interlayer equalization of the microclimate. A purely structural design of such a system is insufficient: the disc motion law, total inertia, load reflected to the motor shaft, acceleration and deceleration torques, and the transient behavior of the electric drive must be determined in an interrelated manner.

The objective of the study is to formulate a kinematic-dynamic model of the rotating-disc mechatronic system, integrate its mechanical and electrical subsystems into a unified electromechanical model, derive the corresponding transfer function, and analyze the dynamic characteristics of the drive. In each cycle, the discs are intended to rotate through half a revolution, i.e., $\varphi = \pi$ rad, and remain stationary during the rest of the cycle. This cyclic algorithm enables the motor to operate only during repositioning and thereby contributes to lower energy consumption than continuous rotation.

LITERATURE REVIEW. In the modeling of mechatronic drives, Newton-Euler equations constitute the principal theoretical framework for the mechanical

subsystem, whereas Kirchhoff's laws are applied to the electrical subsystem. The speed and position models of a DC motor make it possible to relate armature resistance and inductance, torque constant, back-electromotive-force constant, rotor and load inertia, and the friction coefficient within a unified system of equations [1–4]. In control theory, the Laplace transform is used to obtain transfer functions from these differential equations and to evaluate system time constants, poles, transient response, and stability [1, 3].

For rotating mechanisms, the load moment of inertia is reflected to the motor shaft using the equivalence rule associated with the square of the gear ratio. The parallel-axis theorem is also applied when calculating the inertia of supports and payloads located at a finite distance from the central shaft [2, 5]. This approach is appropriate for the rotating-disc incubation mechanism because the egg trays rotate together with the discs and their mass directly affects the total dynamic load.

Unlike conventional theoretical treatments considered separately, this study integrates the periodic 180° motion cycle, a trapezoidal velocity profile, the contribution of the payload to the total inertia, and the electromechanical characteristics of the motor within a single computational model. Consequently, a consistent relationship is established from the mechanism geometry to the required motor torque and dynamic response.

RESEARCH METHODOLOGY

The study employs analytical modeling, classical mechanics, electric-drive theory, and automatic-control methods. The computational object is assumed to consist of horizontal discs mounted at equal intervals on a vertical central shaft, connecting supports, payload placed on the discs, and a DC motor coupled through a gear transmission. In the structural configuration, temperature sensors positioned at different disc levels enable layer-by-layer monitoring of the microclimate.

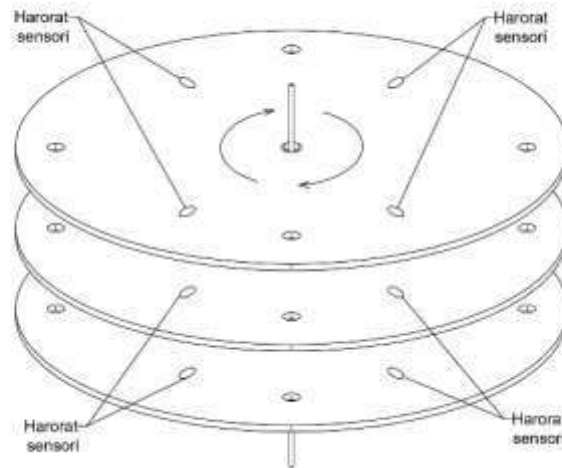


Figure 1. Structural configuration of the rotating-disc mechatronic system

Note: temperature sensors are positioned at multiple disc levels for layer-wise microclimate monitoring.

The principal model parameters are the disc radius R , number of discs n_d , disc thickness h_d , disc-material density ρ_d , mass of one disc m_d , spacing between discs H , shaft radius r_o , shaft length L_o , shaft-material density ρ_o , shaft mass m_o , number of supports n_t , radial distance of the supports from the shaft r_t , mass of one support m_t , number of trays N_q , mass of one tray m_q , total tray mass $M_q = N_{qm}_q$, rotation angle ϕ , cycle period T_s , and gear ratio i .

Table 1. Principal operating and calculated parameters used in the analysis

Parameter	Symbol	Value	Description
Rotation angle	Φ	π rad (180°)	Disc rotation per cycle
Total motion time	T	15 s	0-2 s acceleration; 2–13 s constant speed; 13–15 s deceleration
Maximum disc profile speed	$\Omega_{\max}$	0.2417 rad/s	Trapezoidal kinematic profile
Mechanical time constant	TM	27.7 ms	Simplified speed channel
Steady-state speed under step input	Ω_{ss}	1.488 rad/s	Step-response value of the electromechanical model

Maximum calculated torque	$T_{\max}$	$\approx 0.221 \text{ N}\cdot\text{m}$	Maximum value during acceleration
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The motion law is described by the angular position $\varphi(t)$, angular velocity $\omega(t)$, and angular acceleration $\varepsilon(t)$. During the rotation phase, for an output-shaft speed of n rpm, the nominal angular velocity is determined as follows:

$$\omega = \frac{\pi n}{30} \quad (1)$$

Under the constant-speed approximation, the time required for the disc to rotate through a prescribed angle φ is determined from $t_b = \varphi/\omega$. During start-up, over $0 \leq t \leq t_a$, the acceleration is assumed constant and is expressed as $\varepsilon = \omega/t_a$, with $\omega(t) = \varepsilon t$. During deceleration, the angular acceleration is negative. Thus, the velocity profile is formed as a three-segment trapezoidal function[5].

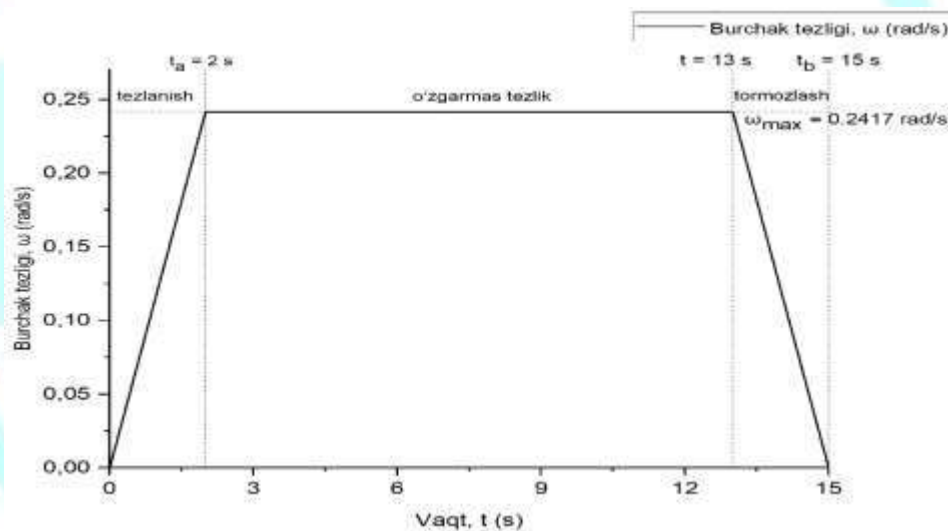


Figure 2. Trapezoidal angular-velocity profile of the rotating disc

Profile phases: acceleration (0–2 s), constant speed (2–13 s), and deceleration (13–15 s); $\omega_{\max} = 0.2417 \text{ rad/s}$.

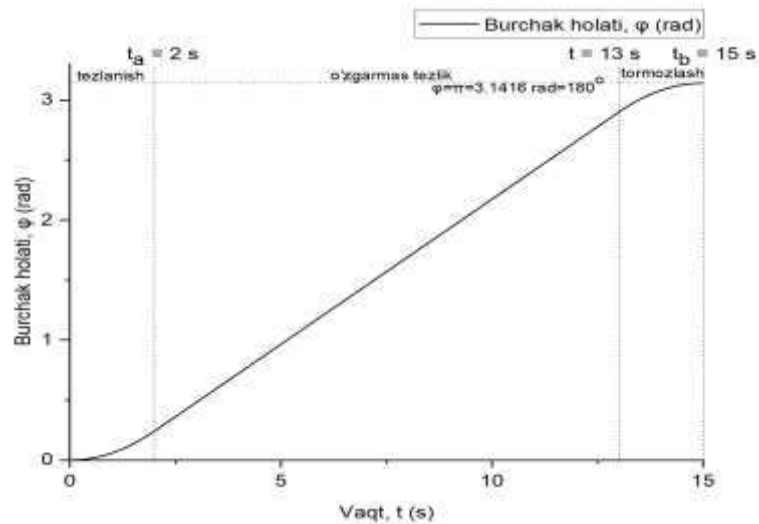


Figure 3. Variation of the disc angular position with time

The angular position reaches $\varphi = \pi \text{ rad}$ (180°) at $t = 15 \text{ s}$.

For the dynamic model, the moments of inertia of all rotating elements about the central axis were summed. The contributions of the discs, shaft, supports, and payload were included in the total inertia J_Σ . The parallel-axis theorem was applied to supports and loads located away from the center. Because a gear transmission is used, the load inertia was reflected to the motor shaft, and the equivalent inertia was expressed in the following general form:

$$J_{eq} = J_{dv} + \frac{I}{i^2} \quad (2)$$

The rotational equation of motion referred to the motor shaft was formulated according to the Newton–Euler law:

$$J_{eq} = \frac{d\omega}{dt} + b\omega + \tau_{yuk} = \tau_{dv} \quad (3)$$

The DC-motor torque was related to the armature current by $\tau_{dv} = K_t i_a$, while the back electromotive force was represented as $e_b = K_e \omega$. For the armature circuit, Kirchhoff's equation is:

$$L_a = \frac{di}{dt} + R_a i_a + e_b = u(t) \quad (4)$$

Assuming zero initial conditions and treating the external load torque as a separate disturbance input, the transfer function from voltage to angular velocity was obtained using the Laplace transform:

$$W(s) = \frac{\Omega(s)}{U(s)} = \frac{K_t}{(L_a s + R_a)(J_{eq} s + b) + K_t K_e} \quad (5)$$

If the electrical time constant is much smaller than the mechanical time constant ($T_e \ll T_m$), the model can be simplified by taking $L_a \approx 0$. The speed channel then reduces to a first-order aperiodic element. Because angular position is the integral of angular velocity, a $1/s$ integrator is introduced into the voltage-to-position transfer function, giving the system Type-1 astatism.

RESULTS. The kinematic calculation divides the 180° rotation cycle into acceleration, constant-speed, and deceleration phases. According to the calculated profile, acceleration occurs over 0–2 s, constant-speed motion over 2–13 s, and deceleration over 13–15 s. The nominal maximum disc speed is $\omega_{max} = 0.2417$ rad/s, while the angular position reaches $\varphi \approx \pi = 3.1416$ rad, or 180° , at the end of the cycle. This confirms that the selected trapezoidal velocity profile provides the specified repositioning angle.

The dynamic response of the electromechanical model exhibits first-order aperiodic behavior. Under a step-voltage input, the angular velocity approaches its steady-state value exponentially. The calculated response gives a mechanical time constant of $T_M = 27.7$ ms, with $3T_M = 83.1$ ms and $4T_M = 110.8$ ms. The steady-state speed is $\omega_{ss} = 1.488$ rad/s; the 95% and 98% levels are 1.414 rad/s and 1.458 rad/s, respectively.

The pole map was used to assess the stability characteristics of the system dynamics. Because the position channel contains an integrator, one pole is located at $s_1 = 0$, while the second pole corresponding to the mechanical mode is located at $s_2 \approx -36.1 \text{ s}^{-1}$. Since the dominant pole of the speed channel lies in the left half-plane, its transient response is decaying and bounded.

The time variation of torque shows that the torque is highest during acceleration, intermediate during steady rotation, and lower during deceleration. For the calculated cycle, $\tau_{\max} \approx 0.221 \text{ N}\cdot\text{m}$, $\tau_{\text{avg}} \approx 0.180 \text{ N}\cdot\text{m}$, and $\tau_{\text{dec}} \approx 0.139 \text{ N}\cdot\text{m}$. Therefore, motor selection should account not only for the steady-state torque but, in particular, for the maximum torque required during start-up.

Under steady-state conditions, the required motor torque is used to overcome friction and the external load; during acceleration, the dynamic component $J_{\text{eq}}\varepsilon$ is added. Accordingly, the selected motor must provide a starting torque no lower than the calculated maximum torque, its rated speed must ensure the required disc speed through the gear transmission, and its supply voltage must be compatible with the power source of the control system. Mechanical power is determined from $P = \tau\omega$.

DISCUSSION. An important feature of the proposed model is that the structural parameters and the dynamic quantities relevant to control are linked within a single computational sequence. Changes in the number or radius of the discs, the arrangement of the supports, or the payload mass first alter J_{Σ} , subsequently J_{eq} , and ultimately the torque required for acceleration. Thus, any modification of the mechanical structure directly affects motor selection and the control algorithm.

The trapezoidal velocity profile is more practical than an abrupt step command: the acceleration and deceleration phases limit mechanical shocks, while the constant-speed interval enables the 180° rotation to be completed within a controlled time. However, the model assumes linear viscous friction, a rigid backlash-free transmission, and a symmetrically distributed load. In a physical device, bearing friction, gearbox backlash, disc deformation, uneven tray placement, and temperature-dependent variation in motor parameters may introduce additional modeling errors.

Reduction of the speed channel to a first-order model is justified when $T_e \ll T_m$. If the motor inductance or the electrical dynamics of PWM control are significant, the complete second-order model should be retained. Because the position

channel contains an integrator, accurate stopping at exactly 180° should not rely solely on open-loop time-based control; in a practical system, feedback from an encoder or another position sensor can improve rotational accuracy. This recommendation follows directly from the astaticism of the model.

The results further indicate that the motor should be selected not solely on the basis of rated power, but by jointly considering maximum starting torque, transient duration, and gear ratio. In cyclic operation, activating the motor only during the rotation phase is favorable for reducing energy consumption; however, an assessment of actual energy efficiency requires separate experimental determination of the efficiencies of the motor, gearbox, and control unit.

CONCLUSION. A kinematic-dynamic model of the rotating-disc mechatronic system was developed, with the mechanical and electrical processes represented within a unified set of electromechanical equations. The periodic 180° disc rotation was described by a trapezoidal profile consisting of acceleration, constant-speed motion, and deceleration. The total inertia of the rotating components and payload was reflected to the motor shaft, and the dynamic model of the drive was formulated on the basis of the Newton–Euler equation.

Combining the mechanical model with the armature-circuit equation of the DC motor yielded a second-order transfer function from voltage to angular velocity and, for a small electrical time constant, its simplified first-order form. The presence of an integrator in the angular-position channel determines the Type-1 astatic behavior of the system. The calculations yielded a 15 s rotation cycle, a kinematic-profile maximum of $\omega_{\max} = 0.2417$ rad/s, a mechanical time constant of $T_M = 27.7$ ms, and a maximum torque of $\tau_{\max} \approx 0.221$ N·m. Pole analysis confirmed the dynamic stability of the speed channel.

The proposed model provides a computational basis for selecting the drive of the rotating-disc incubator mechanism, substantiating the gear ratio, specifying torque and power reserves, and subsequently developing a closed-loop control algorithm.

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